Sinusoidal Word Problems! Yay!

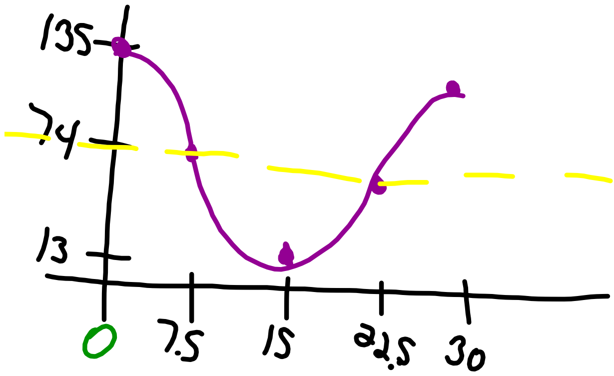
The London Eye, a ferris wheel in London, has a diameter of 122m. At its highest point, it stands 135m above the ground. Each rotation takes 30 minutes. Write an equation to describe the height of a person riding the London Eye over a 30 minutes period of time if they begin:

a. at the top

b. at the middle right

c. at the bottom

d. at the middle left



b (from period) $= \frac{2\pi}{\text{period}}$

amplitude = max − mid = $\frac{\text{max} - \text{min}}{2}$

h (phase shift) where graph starts

k (vertical shift) = midline = $\frac{\text{min} + \text{max}}{2}$

At top:

$b = \frac{2\pi}{30} = \frac{\pi}{15}$ $\quad k = 74$

$a = 61$ $\quad h =$ none

$$y = 61 \cos \frac{\pi}{15} x + 74$$

Middle Right

$y = 61 \sin \frac{\pi}{15} x + 74$

bottom

$y = -61 \cos \frac{\pi}{15} x + 74$

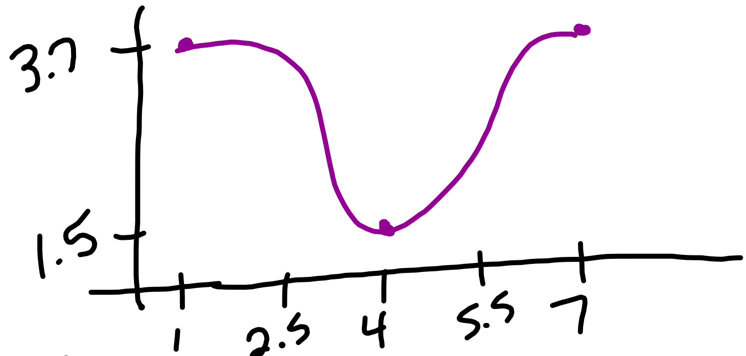
mid. left

$y = -61 \sin \frac{\pi}{15} x + 74$

OIL WELL PROBLEM

2) The jack on an oil well goes up and down, pumping out of the ground. As it does, the distance varies sinusoidally with time. At time = 1 sec, the distance is at its maximum, 3.7 meters. At time = 4 sec, distance is at its minimum, 1.5 meters.

a) Sketch a graph.



pd = 6 sec

$b = \frac{2\pi}{6} = \frac{\pi}{3}$

$a = \frac{3.7 - 1.5}{2} = 1.1$

h = 1 sec $(x-1)$

$v = \frac{1.5 + 3.7}{2} = 2.6$

b) Write an equation.

$y = 1.1\cos\frac{\pi}{3}(x-1) + 2.6$

c) Find the distance when time = 5.5 seconds.

EXTRATERRESTIAL BEING PROBLEM

3) Researchers find a creature from an alien planet. Its body temperature varies sinusoidally with time. 35 minutes after they start timing, it reaches a high of 120° F. 20 minutes after that it reaches its next low, 104°F.

pd = 40

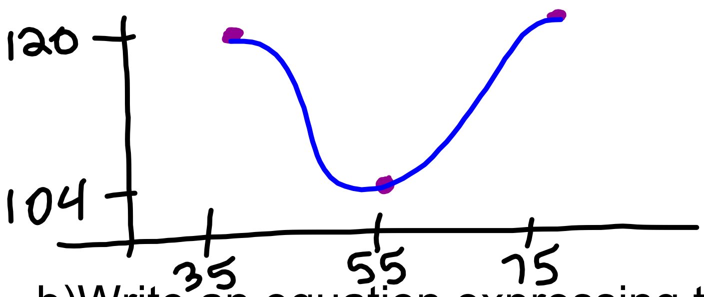
$b = \pi/20$

$a = \frac{120-104}{2} = 8$

$h = 35$ $(x-35)$

$K = \frac{120+104}{2} = 112$

a) Sketch a graph.



b)Write an equation expressing temperature in terms of minutes since they started timing.

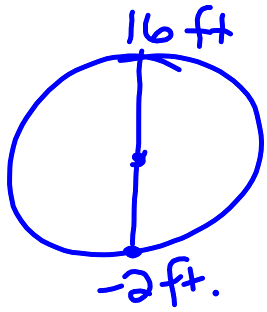
$y = 8\cos\frac{\pi}{20}(x-35)+112$

c) What was the temperature when they first started timing?

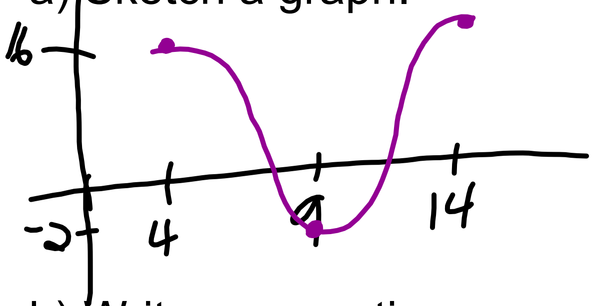
$y = 8\cos\frac{\pi}{20}(0-35)+112 = 117.7°$

Steamboat Problem

4) Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, he noticed a dead fish caught on one of the paddles. As the wheel turned, the distance, d, that the fish was from the water's surface was a sinusoidal function of time. When his stopwatch read 4 seconds, the fish was at its highest, 16 feet above the water's surface. It took another 10 seconds before the fish reached that height again. The diameter of the wheel was 18 feet.



a) Sketch a graph.



pd = 10

$b = \frac{2\pi}{10} = \frac{\pi}{5}$

$a = \frac{max - min}{2} = \frac{16 - {}^-2}{2} = 9$

$k = \frac{min + max}{2} = \frac{16 + {}^-2}{2} = 7$

b) Write an equation.

$y = 9\cos\left(\frac{\pi}{5}(x-4)\right) + 7$ $h = 4 \rightarrow (x-4)$

c) How far above the surface was the point when Mark's stopwatch read 5 sec? 17 sec?

14.28 ft 4.22ft

BUOY PROBLEM

5) If the equilibrium point is y = 0, then
models a buoy bobbing up and down in the water where time is in seconds.

a) Where is the buoy at t = 0?   at t = 7?

b) What is the maximum height of the buoy?   the minimum?

c) What is the period?

Spring Problem

6) A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. (Ignore gravity and assume the spring keeps moving.) You start a stopwatch. When the stopwatch reads 0.3s, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8s.

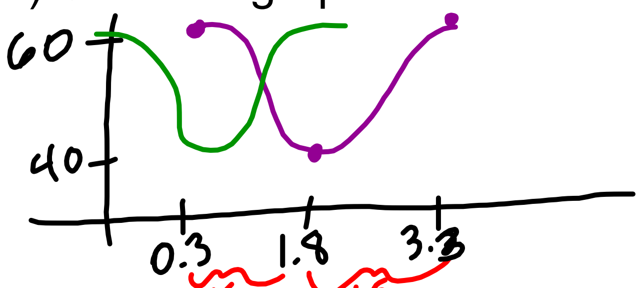
pd = 3 $b = \frac{2\pi}{3}$

$a = \frac{60-40}{2} = 10$

$k = \frac{60+40}{2} = 50$

h = 0.3 → (x − .3)

a) Sketch a graph.



b) Write an equation expressing distance from the floor in terms of the number of seconds the stopwatch reads.

$y = 10\cos\left(\frac{2\pi}{3}(x-0.3)\right)+50$

c) Predict the distance from the floor when the stopwatch reads 17.2 s.

$10\cos\left(\frac{2\pi}{3}(17.2-0.3)\right)+50$

$= 43.31$ ft.

d) What was the distance from the floor when you started the stopwatch?

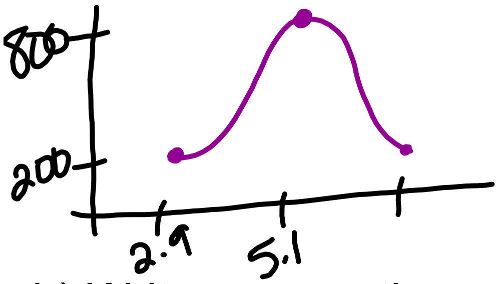
$10\cos\left(\frac{5\pi}{3}(0-.3)\right)+50 = 58.10$ ft.

Fox Population Problem

7) Naturalists find that the populations of some kinds of predatory animals vary periodically. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept when time t = 0 years. A minimum number, 200 foxes, existed when t = 2.9 years. The next maximum, 800 foxes, occurred at t = 5.1 years.

a) Sketch a graph.

* graph is reflected, a must be negative



b) Write an equation expressing the number of foxes as a function of time t.

c) Predict the population when t = 7.

Spaceship Problem

8) When a spaceship is fired into orbit, it goes into an orbit that takes it alternately north and south of the equator. Its distance from the equator is approximately a sinusoidal function of time. Suppose that a spaceship is fired into orbit. After ten minutes it reaches its farthest distance north of the equator, 4000km. Half a cycle later it reaches its farthest distance south of the equator, also 4000 km. The spaceship competes an orbit once every 90 min. Let y be the number of kilometers the spaceship is north of the equator. Let t be the number of minutes that have elapse since liftoff.

a) Sketch a complete cycle of the graph. Write an equation expressing y in terms of t.

b) Use the equation to predict the distance of the spaceship from the equator when t = 25; t = 163

Tide Problem

9) Suppose that you are on the beach at Port Aransas, Texas. At 2:00 pm on March 19, the tide is in (i.e., the water is at its deepest). At that time you find that the depth of the water at the end of the pier is 1.5 m. At 8:00 pm the same day when the tide is out, you find that the depth of t he water is 1.1 m. Assume the depth of the water varies sinusoidally with time.

a) Derive the sinusoidal model for this problem expressing the depth of the water in terms of the number of hours since noon on March 19th .

b) Predict the depth of the water at 4 pm on March 19th.

Health Problem

10) An average seated adult breathes in and out every 4 seconds. The average minimum amount of air in the lungs is 0.08 liter, and average maximum amount of air in the lungs is 0.82 liter. Suppose the lungs have a minimum amount of air at t = 0, where t is the time in seconds.

a) Sketch a graph.

b) Write an equation that models the amount of air in the lungs.

c) Determine the amount of air in the lungs at 5.5 seconds.

Tide Problem #2

11) Burntcoat Head in Nova Scotia, Canada, is known for its extreme fluctuations in tides. One day in April, the first high tide rose to 13.25 ft. at 4:30 am. The first low tide at 1.88 ft. occurred at 10:15 am. The second high tide was recorded at 4:45 pm.

a) Sketch a graph.

b) Write an equation modeling the tides in terms of x, the number of hours since midnight.

c) How high was the tide at 8AM?

d) What is the vertical shift and what does it represent?

e) What is the amplitude and what does it represent?

12) For several hundred years, astronomers have kept track of the number of solar flares, or “sunspots” which occur on the surface of the sun. The number of sunspots counted varies periodically from a minimum of about 10 per year to a maximum of about 110 per year. Between the maximums that occurred in the years 1750 and 1948, there were 18 completed cycles.

a) What is the period of the sunspot cycle?

b) Assume that the number of sunspots varies sinusoidally within the first year. Sketch a graph of two sunspot cycles, starting in 1948.

c) Write an equation expressing the number of sunspots per year in terms of the year.

d) What is the first year after 2000 in which the number of sunspots will be about 35? A maximum?

13) A tsunami is a fast moving ocean wave caused by an underwater earthquake. The water first goes down from its normal level and then rises an equal distance above its normal level, and finally returns to its normal level. The period is about 15 minutes. Suppose that a tsunami with an amplitude of 10 meters approaches the pier at Honolulu, where the normal depth of the water is 9 meters.

a) Assuming that the depth of the water varies sinusoidally with time as the tsunami passes, predict the depth of the water at the following times after the tsunamis first reaches the pier. 1. 2 min. 2. 4 min 3. 12 min.

b) According to your model what will be the minimum depth of the water be?

14) A variable star is one whose brightness alternately increases and decreases. For the most visible star, Delta Cephei, the time between periods of maximum brightness is about 6 days. The average brightness (magnitude) of the star is 4.0 and its brightness varies from a low of 3.5 to a high of 4.5 magnitudes. Express the brightness as a function of time where at t = 0, the magnitude =4.0.

15) A Ferris wheel has a radius of 10 meters, and the bottom of the wheel passes 1 meter above the ground. The Ferris wheel makes one complete revolution every 20 seconds, find an equation that gives the height above the ground as a function of time?

16) A mass is suspended on a spring. The spring is compressed so that the mass is located 5 cm above its rest position. The mass is released at time $t = 0$ and allowed to oscillate. It is observed that the mass reaches its lowest point 0.5 seconds after it's released. Find an equation that describes the motion of the mass.