Wiburg, K., Parra, J., Mucundanyi, G., Latorre, J., & Torres, R. C. (2017). Constructivist instructional design models applied to the design and development of digital mathematics game modules. *International Journal of Technology in Teaching and Learning, 13(1*), 1-15.

Constructivist Instructional Design Models Applied to the Design and Development of Digital Mathematics Game Modules

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Faculty and Doctoral Students in a Learning Design & Technology program have been researching and studying instructional design models over the last year in light of current changes in learning and design theory. This work is part of a yearlong project related to the following two questions: 1) What are instructional design (ID) models that emphasize constructivism, and 2) How can these ID models be applied to the design of digital learning resources, in this case their application to digital math game modules to facilitate constructivist learning? This article is focused specifically on the application of several emerging design models to the development of digital mathematics game modules for two NSF-sponsored Discovery K-12 projects (#0918794, #1503507). The games (Math Snacks) and learning modules in this learning games project are based on researched conceptual gaps in elementary and middle school mathematics and the development of game-based and inquiry models for addressing these gaps.

Keywords: learning design, math games, inquiry, graduate programs in technology, emerging instructional design models, K-8 mathematics

INTRODUCTION

This article was produced by an evolving writing group of faculty and graduate students in a southwestern College of Education, Learning Design & Technology (LDT) program, which highlights constructivist and constructionist models of instructional and learning design. This research was first presented at the 2017 SITE conference (Wiburg, Parra, Mucundanyi, Torres, & Latorre, 2017) and is now being reported in additional detail. The writing group has been studying emerging models of learning design in order to inform our research and practice. The purpose of this article is to examine the relevance of emerging, constructivist instructional design models to the development of a digital games

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development project, Math Snacks: Early Algebra which is funded by the National Science Foundation (NSF #1503507). Included in this article is the background of the study, application of constructivist ID models to practice, a final discussion about applying constructivist ID models to game design, and a conclusion.

The university learning and technology doctoral program from which this article evolved is committed to providing ways for doctoral students to connect theories to practice and offers multiple opportunities for apprenticeships in real world instructional design, teaching, and publication writing. Currently in the second phase of NSF funding, the project staff is designing mathematics games and inquiry-based learning modules that provide reluctant mathematics learners a different way to learn Early Algebra concepts. This paper situates the constructivist ID models we have identified in relationship to our own current design of the latest Math Snacks Learning Modules.

BACKGROUND OF THE STUDY

In this section, the following is discussed: (a) our use of gap research in mathematics learning, (b) previous research about the impact of Math Snacks on student achievements, and (c) the initial learning games design model (LGDM) used for the project.

THE GAP RESEARCH

Before there was Math Snacks, there was extensive research on gaps in learning mathematics in grades 3^{rd}-8^{th}. Researchers analyzed 24,000 standardized tests in their home state, which included at least 40% of open-ended short and long answer items. This was followed by over 500 hours of extensive observations of teachers and students doing the targeted math in classrooms. It became clear from our research findings (Wiburg, Chamberlin, Valdez, Trujillo, & Stanford, 2016; Trujillo, Chamberlin, Wiburg, & Armstrong, 2016) that there were distinct conceptual gaps in elementary and middle school mathematics students' knowledge of specific core mathematics topics and practices. It was surprising to find that these gaps constituted a common pattern across many different demographics in the state, including high- need districts, high English Learner districts, and a few mainstream districts. The areas that were discovered to be troubling for all math students in grades 3-8 included: ratios, fractions, decimals, the number system and understanding number properties. These gaps were addressed in the first Math Snacks project.

PREVIOUS RESEARCH SHOWED INCREASED MATH ACHIEVEMENT

In the previously funded grant, mathematics educators, mathematicians, learning specialists and game developers collaborated to develop and test five games, six animations and related support tools (Wiburg, 2014; Wiburg, Michels, & Savic, 2010). Accompanying teacher guides, learner guides, "Teaching With" videos, and correlations to the Common Core reflect an inquiry-based and constructivist approach. Research was a driving factor throughout the development of Math Snacks materials, including investigations into learners' needs, extensive user testing, and classroom observations. While the achievement levels might have started at lower points in some districts, all students showed the same pattern of lack of mathematics knowledge regardless of socio-economic status, grade level, type of school, or education level of parents.

To test the impact of the first Math Snacks, the research team conducted different pilot and large-scale studies. One of pilot studies was in a low-income, urban school district, where 9 middle school teachers in 3 schools used the animations and related activities. Their students showed significant learning gains, particularly of concepts. Additionally, a large-scale randomized controlled study was performed with 48 fifth grade classrooms in

14 schools in one low-income urban school district in southern New Mexico (Trujillo, Chamberlin, Wiburg, & Armstrong, 2016). Classes were randomly assigned to two groups. Seven hundred and fifty-one fifth-graders participated in this study in 2013-2014. The research involved a Delayed Intervention Model borrowed from medical research. All students in both experimental and control groups took the first test before Math Snacks intervention and had pre-treatment equivalency. During the fall of 2013 teachers and classrooms selected in Group A used Math Snacks and activities with their district-approved curriculum for five weeks, and teachers in Group B only used the district-approved curriculum during the same time. All students took the second test. After a winter break, Teachers and students in Group B then integrated Math Snacks into instruction. After receiving the delayed intervention, Group B subsequently caught up with Group A (Test 3). Finally, all students took the third test. The test used was the Measure of Mathematics Learning II (MMII), developed by the Math Snacks researchers. The result showed that the scores of Group A were significantly higher in gains on the measure of mathematics learning as shown on Figure 1.

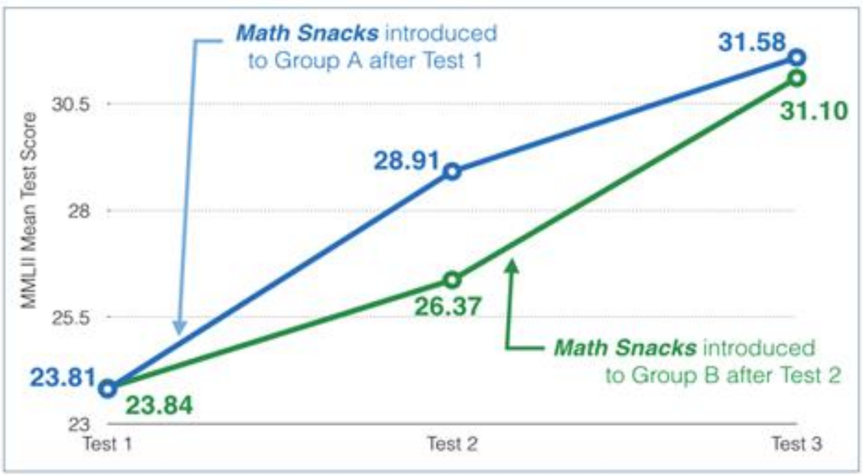

Figure 1. Learning gains of fifth-grade students, Learning Games Lab, New Mexico State University.

This final research demonstrated the effectiveness of the products and showed significant gains in math content knowledge for the students who used them. These gains occurred regardless of student demographics. Females did as well as males, and English-Language Learners (ELL) gained as much as regular students and our majority Hispanic learners did as well as mainstream students in terms of the math gains found. However, a word of caution is needed in interpreting these findings (Wiburg, Chamberlin, Valdez, Trujillo, & Stanford, 2016; Trujillo, Chamberlin, Wiburg, & Armstrong, 2016; Valdez, Trujillo, & Wiburg, 2013). While the gains for all groups were equivalent in the study, some students, who may have had less learning opportunities in their schools, probably started lower than others.

THE CURRENT MATH SNACKS PROJECT

Currently, we are addressing additional conceptual gaps in areas hypothesized to be missing learning opportunities in grades 4^{th}-6^{th} in what is called Early Algebra (Wiburg, 2017). Wiburg suggests three research strands, which describe the transition from elementary arithmetic to symbolic algebra, why it is so difficult, and how this transition should be addressed. This content is being addressed in the new project. However, content is only one of the complexities of this new project. The research team believes in situated cognition as well as the affordances of technology to provide representation as strategies to integrate into our latest design (Wiburg, Chamberlin, Trujillo, Parra, & Stanford, 2017).

A great deal has been learned over the last 30 years about the brain and how people learn (Bransford, Brown, & Cocking, & National Research Council (US.), 1999). Learning theory has developed from a simple behaviorist view to a focus on cognitive learning, and eventually to the inclusion of social construction of knowledge by the learner in collaboration with others. Vygotsky's views on learners' interactions in social and cultural environments have also influenced constructivist and constructionist approaches to learning (Wertsch, 1985, 1998). Some theorists (Papert, 1980, 1996a, 1996b) represent advocates of constructionism, believing that learning works best when students are interacting with some type of artifact, whether an electronic object, a programming environment, or the creation of objects to be manipulated in an online game. We are defining early algebra as structured arithmetic in grades 4^{th}-6^{th} that leads from carefully designed structured number relationships which introduce algebraic thinking and move toward early use of variables as useful (Stephens et al., 2017).

It was because of the difficulty in really applying current early algebra ideas as well as the desire to create a constructivist games and supplemental materials that led to a serious investigation of how constructivist learning can be used in this project. The authors describe a year-long inquiry-practice process intended to answer the questions: 1) What are instructional design (ID) models that emphasize constructivism? and 2) How can these ID models be applied to digital learning resource design, such as Math Snacks to facilitate constructivist learning?

INITIAL LEARNING GAMES DESIGN MODEL

At the time of our initial funding for Math Snacks in 2007, there was "minimal research on game development" and the design team members created the *Learning Games Design Model* (Chamberlin, Trespalacios, & Gallagher, 2014, p. 153). The *LGDM* is composed of three phases: pre-development, development, and final stages (Chamberlin, Trespalacios, & Gallagher, 2014; Chamberlin, Trespalacios, & Gallagher, 2012). All team members in the Learning Games Lab were involved from the beginning to the end of the creative process for design and development; playing roles in multiple formative assessments of the materials by students in the Learning Games Lab, in partner teacher classrooms, and in the Math Snacks summer camps. Figure 2 (see next page) is a graphic illustration of the initial Learning Games Design Model (*LGDM*.)

DESIGNING GAMES VERSUS DESIGNING FOR CONSTRUCTIVIST LEARNING

There has been extensive writing about different designs for building games and digital courses (Willis, 2009) which range from old-fashioned top-down linear design based on learning objectives to more flexible designs that allow for iterative design by cross-disciplinary researchers. The LGDM is very much a model that allows for extensive exploration and testing within each stage of the model and expects many changes over time. Researchers, faculty, and doctoral students are involved in exploration of current games in the content area and have opportunities to explore and learn the math content as well as play with different parts of games. This is a good model for the design of games.

However, not a lot has been written about what needs to be designed into games so that learners can participate in constructivist learning experiences. How can a game be designed so that it facilitates choices for students in activities, allows multiple retries and even supports multiple different ways to solve problems as part of the game? Making constructivist games is not easily adapted to digital media and we started a research and writing group to review literature and see what emerging models in constructivist design might be useful for our design work and our move toward constructivist design for learning.

Graduate student researchers offered to complete an extensive review of the literature on emerging design models. Their study of the models and their applications to practice are reported below.

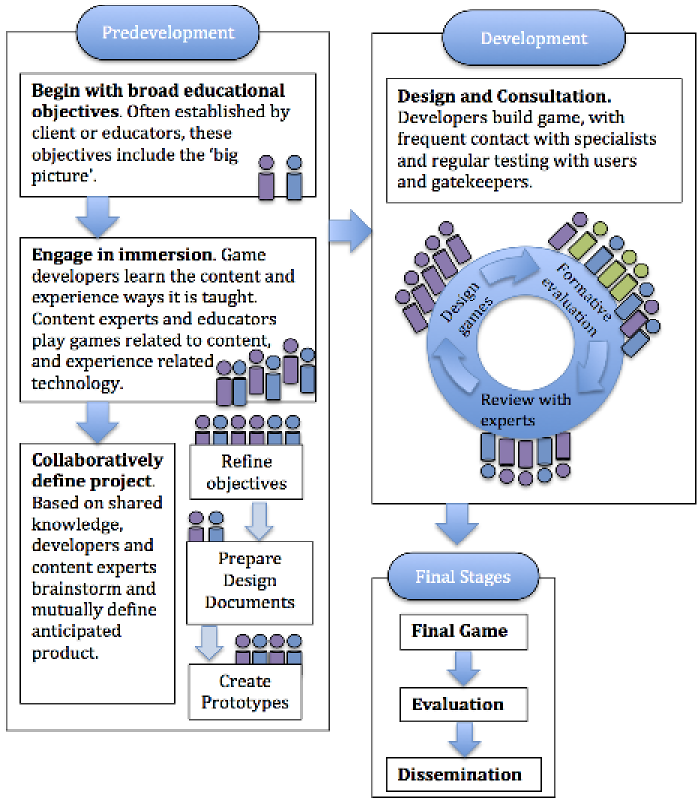

Figure 2. Learning Games Design Model, Learning Games Lab, New Mexico State University.

APPLYING CONSTRUCTIVIST ID MODELS TO PRACTICE

The authors describe a year-long inquiry-practice process intended to answer the questions: 1) What are instructional design (ID) models that emphasize constructivism?, and 2) How can these ID models be applied to digital learning resource design, such as Math Snacks to facilitate constructivist learning? To answer the above questions, the authors used literature reviews, in-depth discussions of relationships between digital design and learning in a seminar, and their individual as well as collaborative design work with digital game design.

The constructivist ID models identified, discussed and applied as *Design Perspectives* in this paper include: *Recursive, Reflective Design and Development model (R2D2)*; *Eternal, Synergistic Design Model*; *The Layers of Negotiation model*; and *Appreciative Instructional Design (AiD).* In the following section each of these innovative, constructivist/constructionist models is considered in light of what the models can contribute to design work both in the Math Snacks project and as illustrations for possible further work in digital learning design by faculty and doctoral students enrolled in a program in Learning Design and Technology at a Southwest Border University. A section is also added at the end of this section to conclude this team's discussions about applying constructivist instructional design models to practice.

THE RECURSIVE REFLECTIVE DESIGN AND DEVELOPMENT MODEL (R2D2)

One of the earliest attempts to build a constructivist instructional design model was the Recursive method for instructional design by Willis (1995) who focused specifically on the process of design by the designers, rather than how the design effects learners. Willis defines Constructivist-Interpretivist Instructional Design Model as a process, and then defined this process as a model called Recursive Reflective Design and Development (R2D2) that has the following characteristics:

1. The design process is recursive, nonlinear, and sometimes chaotic. 2. Planning is organic, developmental, reflective, and collaborative. 3. Objectives emerge from design and development work. 4. General ID experts do not exist. 5. Instruction emphasizes learning in meaningful contexts (the goal is personal understanding within meaningful contexts). 6. Formative evaluation is critical. 7. Subjective data may be the most valuable. (as cited in Tam, 2009, p. 70).

The initial R2D2 model was one of the first to lay out in some detail a process for creating instructional materials that was based on constructivist theory and applied to digital designers specifically. As Willis mentions, R2D2 is one-way of designing instruction through a process that is collaborative, recursive, and emergent. The design is also non-linear, meaning that any aspects of the design, which are not fundamentally required to be sequential, can be done in any order, as well as revisited and revised at any time. The functional ideas undergirding the R2D2 model are encapsulated in the name by Willis (2009):

1. *Recursive (Iterative), Nonlinear Design* – the steps taken in design do not need to follow a linear or waterfall sequence. The design problem should be able to be accessed from any angle, at any time, any number of times, in any order. The situation or context of how the design is working in practice determines the necessary steps, not the domination of an expert designer.
2. *Reflective Design*– This is a continual cycle of framing the problems that occur in the design process, improvising a solution and finally implementing the solution. The arts, of problem framing, implementation, and improvisation, make up a continuous reflective practice.
3. *Participatory Design* – all stakeholders, including learning design students, need to be included in every aspect of the design process. The users are also involved in continuous design feedback, not as observers from the sidelines or objects to be studied. The collective knowledge on participant feedback can be applied to produce more meaningful instruction.

Willis was focused primarily on the process of design and the role of stakeholders in the design process. Figure 3 illustrates three phases of R2D2.

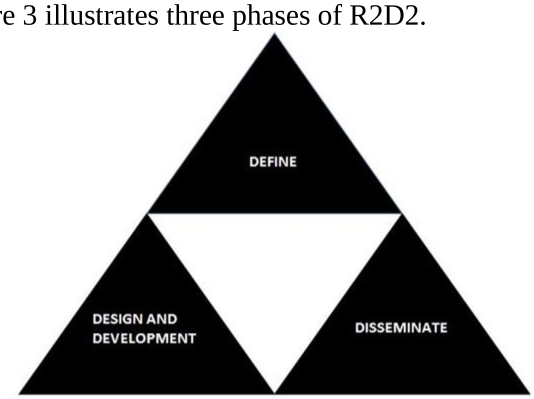

Figure 3. Phases of R2D2, Learning Design and Technology program, New Mexico State University.

Design Perspective 1- Increasing the Participation of Additional Stakeholders including Learners and Teachers

From the practice perspective and in the pre-design and early design phase of a second round of Math Snacks digital design, the current learning design team has incorporated multiple stakeholders throughout the process. In summary, using the Learning Games Design Model with graduate student participation, content experts, learning researchers, and multimedia educational design experts is working to ensure that each aspect of the product meets the criteria for all three of the following: 1) the content is correctly developed for mathematics learning; 2) the process of using the products includes student engagement, constructivist learning, inquiry teaching and culturally-responsive approaches; and 3) there are opportunities for extensive formative evaluation at every stage of the design followed by design-based research on how the product is working in terms of what are students learning, and how are they learning this. A final research study will be done in the last year of the grant, 2019, in multiple classrooms new to the Math Snacks Early Algebra Modules.

Our expansion of the model includes K-12 teachers and students in our targeted grades 4th-6th. Students also present what they are learning to their parents and friends in annual Math Snacks Summer Camps. Teachers share with us at the beginning how they teach the core math concepts we are targeting in our design. They tell us where students have difficulty in learning the concepts and when they have trouble teaching the concepts. They then test different stages of the designed games to see what students are learning from them, what they don't learn, and how the research team might design additional inquiry into the modules to facilitate students' learning of the concepts after or during the game play. We are also working on additional digital embedded assessments and feedback for learners in the Math Snacks Modules.

Finally, we are interested in studying the classroom-learning environment as the new resources are implemented in classrooms. The educational research team is doing formal observations using the *Observation of Learning Environment 2* (OLE2) which has been used in previous NSF projects and measures the quality of a constructivist mathematics environment (Kinzer, Virag, & Morales, 2011).

ETERNAL, SYNERGISTIC DESIGN MODEL

A similar but longer approach to Willis's R2D2 is described by Crawford (2004) as an *Eternal, Synergistic Design* Model. Caroline M. Crawford wanted to develop an e-learning course and did not find instructional models that fit her team's needs. Thus, she decided to design the *Eternal, Synergistic Design Model* (Crawford, 2004). In this model, the analysis phase is integrated into both the design and development phase, and connected to continuous evaluation and feedback. When using this model the product is always under constant revision in response to feedback from users. According to Crawford (2004), "this non-linear instructional design model is paramount, *as the underlying assumption is that the product is never 'final', but merely in a stage of evaluation*" (p. 417). Additionally, when put into practice, this model easily interacts with the Pareto principle of 80:20, 80% satisfaction and 20% dissatisfaction. The design team must wait for 20% of dissatisfaction in complaints or problems identified by users, who use products, in order to analyze those products and make new products that respond to users' needs. The Crawford model is as non-linear as Willis's (1995) model, but the current model focuses more on evaluation and feedback from users or customers to improve products.

Design Perspective 2- Seeing the Developing Math Snacks Home Web Site as a Long-Term Project

The team in the Learning Games Lab has to constantly revise the Math Snacks Home site (*http://mathsnacks.org*). Funding is currently set-aside for this purpose. Educational practices, testing, requirements, and standards, change frequently so the Web Designers in the Learning Games Lab have to constantly revise the Math Snacks Home site. In addition the team is able to respond with modification of the Math Snacks Modules in response to problems, changes in operating systems, changes in application support by browsers, new programming environments, and development of embedded assessment items. The web designers and programmers are also able to respond to inspirations from the team testing the modules in the Games Lab, in a new Games Evaluation Center in the College of Education and in the schools and camps to provide clearer or new information to users of the web site. When the longer term impact of Math Snacks is being considered, technical data on downloads, especially downloads of certain products and games in Spanish and English are continuously analyzed.

THE LAYERS OF NEGOTIATION MODEL

Cennamo (2009) designed the *Layers of Negotiation Model* based on her experience of working on projects intended to create constructivist learning environments for science and mathematics education. The design in those projects did not follow the traditional instructional design in relation to limited and specific learning objectives. Learning objectives can constrain a truly constructivist model and Cennamo, somewhat controversially notes, “constructivism implies that it is inappropriate to set learning objectives for the students” (2009. p. 358). She views instructional design as “a process of constructing knowledge, involving reflection, examining information at multiple times for multiple purposes, and socially negotiating of shared meanings” (p.369). Therefore, the *Layers of Negotiation Model* focuses on identifying critical areas that need negotiation and illuminating the nature of a recursive model. The model emphasizes participatory design, social negotiations, and values *cultural knowledge* of individuals on the design team. The design is iterative, spiral fashion, proceeding through analysis, design, development, and evaluation as each component of instruction is developed.

Cennamo (2009) recommends that activities, assessment and goals must be aligned and enhance cognitive thinking. She provided the following sample questions for designing each instruction: “*1. What kinds of changes in thinking do we want to occur? 2. How will we know if these changes have happened? 3. How will we provide opportunities for learners to examine such issues?*” (p. 365). This model can be seen as related to a common call for serious thought and discussion of what the final learning will look like as well as how we will assess that learning. (See the *Understanding by Design* approach for curriculum design that integrates assessment as critical for student learning and closely related to the goals of the learning (McTighe & Wiggins, 2011)

Design Perspective 3- A Conversation about Student Learning and Assessment

This perspective lead support to our desire as constructivist designers to find other ways to understand what students know and don’t know and especially how they think about the mathematics they are doing. At the point we were talking about this design we were not happy with testing and testing results we had been getting. At the request of the funding agency, researchers spent significant amounts of time in designing an assessment instrument for evaluating learning in the three areas of mathematics being addressed. Core topics developed based on gaps in learning and previous research include: 1) patterns and relationships between co-varying quantities, 2) an understanding of the properties of operations, and 3) how to read and write expressions and equations. Modifying some release items from standardized tests and developing our own items, led by our Lead Mathematician and Testing Specialists, resulted first in over 80 items consisting of multiple-choice and short and long answers related to the three targeted areas. After asking over 100 students to respond to the first 46 items we found that we had indeed targeted some aspects of Algebra learning that the students found difficult. The items measuring these areas of understanding had the lowest scores on this practice test. We also found that scores on the more complex open-ended items were the most predictive of scores on the over-all test, a high score in an open-ended item correlated to a high score on the overall test.

Researchers continued to test items at our annual Summer Math Snacks Camp, June 12-16, 2017. However, we received a strong message from our 4^{th}-6^{th} grade students who came to camp after a long school year of testing ready to resist anything that looked like tests. We had developed short 15 minute *papers* with 4-6 questions for them to take, as part of three different fun classes during the first day, however the kids smelled a test. The resistance to testing has probably been exacerbated by frequent and lengthy testing in their schools as part of the *The Partnership for Assessment of Readiness for College and Careers (PARCC*) which is used in our state. Some students refused to take the test at all, and some students simply left pages or items in the final test unanswered.

As a result of our disappointment in testing and our true desire to understand what kids are learning and how they are learning them, we came up with some new ideas. In our first game which is about writing expressions, we decided to record all the expressions the students typed so we can maybe learn something about how the student was thinking about expressions. We began to talk about capturing student performance during the game as well as putting some kind of short embedded assessment in the game as well. We are currently doing some hands-on activities after the games such as using blocks to build a bridge to try and find out what students are thinking about the game and are they thinking math. It seems necessary as we move toward constructivist learning we will need to develop new forms of assessment.

APPRECIATIVE INSTRUCTIONAL DESIGN (AiD)

Reflected in traditional models of instructional design is a behaviorist paradigm; described by Gustafson and Branch (1997) to be "observed, measured, and planned for in reasonably valid and reliable ways" (p.73). Therefore, emphasis has been placed on models that ensure efficiency in human learning, instruction, and performance. Unfortunately, this is meeting small learning objectives that are easily measured, and does not reflect today's learning environment where there are greater opportunities and demands for continuous changes in relationship to the larger view of complex tasks, at school and at work. New knowledge of learning suggests a more constructivist perspective including support for inquiry, as an important tool for facilitating learning (Bransford, Brown, Cocking, & National Research Council (US.), 1999). As a result of this paradigm shift, instructional design models are "reflective of a learner who can think holistically, evoking and co-creating reality(s)" (Norum, 2009, p. 424).

The designer describes this new model as follows: AiD is grounded from a theoretical foundation in Appreciative Inquiry (AI). Appreciative Inquiry is described as searching for "the best what is rather than looking for exactly what is wrong or needs to be fixed" (Norum, 2009, p. 425). AI gives us a structure for searching out the "goodness" in the system; appreciating "what is" or what "could be" in opposition from a deficit-based approach. When AI is applied to a system, the goal is to discover the factors present when the system is operating at its "best of" level; amplifying those competences needed to perform at the "best of" level thus giving the organization "more" of what it wants (Norum, 2009, p. 425). The question to be asking is what does it look like (resembles) when a system is operating at its best of level? (Norum, 2009).

The AiD model is focused on inquiry-based learning. Since inquiry is at the heart of this model, a fair amount of time must be devoted to modeling query as open-ended in the materials provided, as well as offering spaces for self-reflection. A perspective of a contemplative habit of mind is quintessential to the learning outcomes because the goal for such instruction is to pose questions in order to create additional thinking and dialogical exchanges between learners. Such an exchange incites curiosity about a subject, while making connections to an expanded circle of previous ideas. As a consequence, students are tasked to challenge or make sense of their ideas; such exploration allows students to design their activities, sort out information and apply skills of processing to evaluate their evidence. When students witness the value and importance of their inquisitiveness and intuitive nature, as well as their peers' knowledge contributions, they develop self-confidence and a desire to seek ownership in the learning process, simply because they have determined their role/participation as active learners.

Design Perspective 4- The Game by Itself Is Not Enough

Designing constructivist tools and resources requires decisions about how to integrate inquiry and reflective opportunities into the product design. In the math digital game design project, we include specific questions for teachers to ask after the students have had time to explore and play the game on their own. Our games are designed to be used at various times during the lesson. Teachers or instructors often allow students to play and explore the game or watch a Math Snacks video at the beginning of a lesson on such topics as algebraic patterns or number expressions to allow time for reflection, discovery and conversation between students before the teacher introduces the core math concept. The students then use the games again at the appropriate later times during math class or after school. Games are available for use during or out of school time via the web.

There are suggested protocols for inquiry before, during and after game play. Suggestions include how to stop the students when they seem to be having challenges while playing the game and ask questions like: Well, what have you tried so far? What worked and what didn't work in the game? Or simply stopping the game after 10 minutes to ask the class members to share the strategies they are using to play the game. One of the innovations in the first Math Snacks project was to include videos of teachers teaching with Math Snacks in real classrooms. Many teachers are reluctant to not tell the students the answers. They need to see models of teachers using inquiry in classrooms.

An example of further inquiry-based teaching strategies is attached to one of the designed animations, when students are asked to use a king's feet and a queen's feet to measure a variety of articles in the classroom and then compare the number of feet. This activity is very easy to extend into the discovery of a linear equation regardless of the size of the foot (Uribe-Flórez & Trespalacios, 2013). Another example is that after playing one of the designed games that uses x and y coordinates (Game Over Gopher), students go on to create their own treasure maps using the locations of points on an x and y coordinate map. Applications of the core concepts targeted in the game are provided in teacher or tutor guides and student learning guides. Many of the after game activities are problem-based group activities as suggested above, measuring objects with King and Queen Feet or creating treasure maps in pairs to apply and practice X and Y coordinates.

FINAL DISCUSSION ABOUT APPLYING CONSTRUCTIVIST ID MODELS TO GAME DESIGN

In summary, each of the constructivist ID models applied to developing game-based digital modules and resources provided valuable considerations for the Learning Games Design Model. Researchers relate the R2D2 to what Math Snacks project does in pre-design and early design. This is evident in how the Math Snacks project involves teachers, students, game designers, and researchers in all phases of the game development, pre-development, development, and final stages. Thereby, the Math Snacks project creates a learning experience environment where everyone's input is valued. The relationship of the Eternal, Synergistic Design Model can be similarly relevant pre-design and early design but is also relevant as supportive in the final stages of the Math Snacks project after educational games are available to schools. This is evident with the Math Snacks website that provides additional support to teachers, students, and parents. Feedback on the website from post educational games delivery becomes one of the inputs for taking games design to the next level. The Eternal, Synergistic Design Model requires 20% of dissatisfaction in order to update or change existing design, however, incorporated with LGDM, 20% is not a necessary indicator and designers continuously do research and improve a game design as needed. The Layers of Negotiation model provided a focus for identifying critical areas and providing sample questions that can guide design. Similarly, the Math Snacks game begins with broad educational objectives and learning assessments. Finally, application of Appreciative instructional Design (AiD) provided focus on the collaboration between teachers and Math Snacks team in developing learning objectives, assessment tools, and Math Snacks modules to support educational games.

Authors recommend three additional areas relevant for final considerations regarding the application of constructivist instructional design models to practice specific to game design and the Math Snacks project: 1) instructions and feedback, 2) embedded data collection, and 3) doing things differently.

INSTRUCTIONS AND FEEDBACK

When making decisions about constructivist learning design, it is important to think about what concepts you want the students to discover in the game and follow-up learning activities and what you need to give to the students with the game such as specific instructions and feedback. This is especially important in mathematics because in this content area there are times that rules, like the order of operations, just need to be accepted as rules. However when students discover the rules after playing an expression writing game about the orders that seem to work for expressions they will remember the rule for the order of operations. They will have lived the rule in order to obtain objects in the game that they want. Teachers know that not every activity needs to be "discovered" by students and that lessons and materials often need to include information that they want the students to know in order to learn the desired concepts. This is part of the art of constructivist learning design. In making decisions about learning design you have to consider what aspects of the content you are sharing, and what part of the design will lead the students to construct their own understanding.

EMBEDDED DATA COLLECTION

As a result of all of the Math Snacks Camp personnel having negative experiences with testing, the Lead Mathematician, Dr. Ted Stanford suggested building on the use of embedded data collection. In the past round of math snacks, embedded assessment was used - data collection built directly into the games - to help research the effectiveness of the games. For example, we were able to analyze how much time a student spent playing the game, and how much time it took them to progress from one level to the next. In our proposal for the current funded grant, we proposed using embedded data collection in a similar way, as one of a suite of measures to help judge the effectiveness of the games for student learning.

We feel it would also be valuable at this point in the project to take embedded assessment to a higher level, not just to evaluate the games, *but also to learn more about how students think about basic early algebra concepts.* We would like to invest more resources in collecting data through the games, and in analyzing that data. This change is based on our work on the grant so far. From our study of the literature, and from our own research thus far, we feel that there needs to be more basic research done on how students come to understand and be able to use early algebra concepts. We have also observed, through some of our pilot testing and student observations at Math Snacks Camp, that different students approach and make sense of basic concepts in different ways, and we believe that it would be valuable to further document these differences.

For example, writing numeric expressions is a large part of one of our three key concepts in this project. Current standards focus on this in fifth grade, but many middle school students still struggle with correctly conceptualizing and writing numeric expressions. Based on watching students play one of our prototype games at the camp and in the games lab, we believe it would be a valuable contribution to the research literature to understand how different students' progress from being able to write "4 - 1", to being able to write "(4 - 1) x 2", to being able to write "(4 - 1) x (5 - 3)" (Note that our goal is for students to generate such expressions themselves, not just to evaluate expressions given to them.) We believe we can do this by recording all the expressions that a student attempts to write while playing the game, whether syntactically correct or not, in the order that they attempt them, and then analyzing that record of attempts. Ted is currently studying the sequences of student expressions and learning which they use as part of the new forest builder design. In order to assist us with understanding ongoing student learning, we now have embedded assessments in the first digital game on Expressions, which can track student responses in a systematic way so we can discover the different pathways students might take while playing with making expressions.

DOING THINGS DIFFERENTLY

Since the beginning of the Math Snacks Module design process the researchers have kept in mind the need to design digital products differently in order to facilitate constructivist learning. In order to support students as they try different solutions to problems, the game and the inquiry materials must be designed to facilitate application and group work. This is a challenging task for all game designers today.

CONCLUSION

One of our goals as a Learning Design and Technology program is to provide an exemplary graduate learning experience for our students in which they participate in our classes as part of the design of class goals and activities. They also participate in publication and games testing and development groups as part of their doctoral study. In addition, they contribute to the digital design of connected projects aimed for public access. Instead of writing for the teacher, students have the opportunity to write to authentic audiences and contribute to practice and problem solving. Dr. Julia Parra, one of the authors, has used a framework called *Participatory Class Design* in her classes since 2012 (Abdelmalak, 2013). According to Abdelmalak, five strategies are used for participatory course design including: "1) articulating the rationale of student involvement in curriculum design, 2) pre-determining a set of procedures, 3) brainstorming in small groups, 4) negotiating as a whole class, and 5) facilitating and guiding throughout the entire process" (Abdelmalak, 2013, p. 85). This article is a result of participatory design through a seminar that was initially requested by graduate students to better understand learning theories and instructional design. This article is a product of two semesters of study and writing, as well as summer work, often on our own times, to complete an article on emerging design models, which are very relevant to a real NSF-funded Design and Development Project. This article basically points a direction for designers to consider and explore emerging instructional designs, which have relevance for constructivist learning.

ACKNOWLEDGEMENT

This research was supported, in part, by the National Science Foundation (NSF) grant, award number # 1503507, Math Snacks Early Algebra: Using Games and Inquiry to Help Students Transition from Number to Variable. The Early Algebra grant was preceded by an earlier Math Snacks grant (# 0918794) in game design and inquiry for addressing challenging areas in mathematics in middle schools. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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